

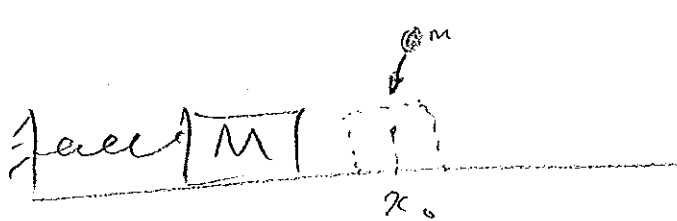
Closed book. No calculators are to be used for this quiz.  
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

A block of mass  $M$  attached to a horizontal spring with force constant  $k$  is moving in simple harmonic motion with amplitude  $A_1$ . As the block passes through its equilibrium position, another block of mass  $m$  is dropped from a small height and sticks to it. (a) Find the new amplitude and period of the motion. (b) Repeat part (a) if the second block is dropped onto the first block when it is at one end of its path.



$$\omega_1 = \sqrt{\frac{k}{M}}$$

$$\text{at } x_0 \quad v \leq v_{\max} \Rightarrow v_{\max} = A_1 \omega_1 = A_1 \sqrt{\frac{k}{M}}$$

we have a completely inelastic collision at drop moment so  $P_i = P_f$

$$M v_{\max} = (M+m) v'_{\max} \Rightarrow v'_{\max} = \frac{M}{M+m} v_{\max} = \frac{M}{M+m} A_1 \sqrt{\frac{k}{M}} = \frac{A_1 \sqrt{kM}}{M+m}$$

we will use conservation of Energy after collision

$$\frac{1}{2} (M+m) v'_{\max}{}^2 = \frac{1}{2} k x_{\max}^2 = \frac{1}{2} k A_2^2$$

$$A_2^2 = \frac{M+m}{k} A_1^2 \frac{kM}{(M+m)^2} \Rightarrow \boxed{A_2 = \frac{M}{M+m} A_1}$$

$$\boxed{\omega_2 = \sqrt{\frac{k}{M+m}}}$$

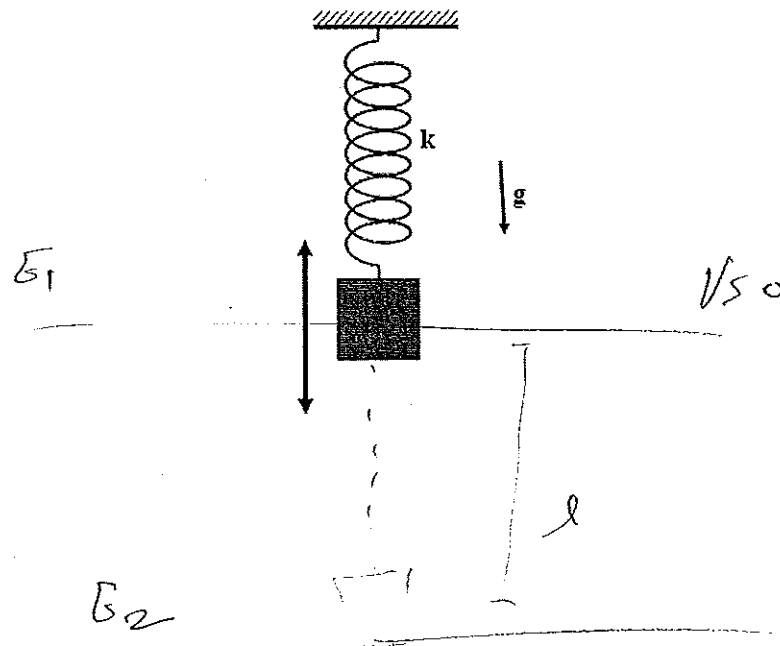
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A spring with force constant  $k$  is hanged in the vertical direction. The spring has negligible mass and it remains initially at rest at its unstretched position. A point particle with mass  $m$  is attached to the free end of the spring with no initial speed. Use energy conservation to find the amplitude of the resulting oscillations



$E_1$  &  $E_2$  Conservation of energy

$$0 = -mgl + \frac{1}{2}kl^2 \Rightarrow mgl = \frac{1}{2}kl^2 \Rightarrow \boxed{l = \frac{2mg}{k}}$$

The Amplitude:  $A = \frac{l}{2} = \frac{mg}{k}$

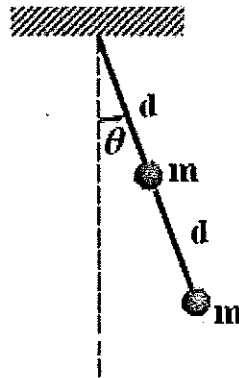
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A physical pendulum formed by a rigid body having two point particles each with mass  $m$  swings around the vertical axis, as shown below. You can ignore the mass of the wires forming the rigid body. Find an expression for the period of small oscillations that will be performed by this physical pendulum.



$$I = md^2 + m(2d)^2$$

$$d_{cm} = \frac{md + 2md}{2m} = \frac{3}{2}d$$

$$T = 2\pi \sqrt{\frac{I}{Mgd_{cm}}}$$

$$T = 2\pi \sqrt{\frac{5md^2}{2mg \frac{3}{2}d}}$$

$$T = 2\pi \sqrt{\frac{5d}{3g}}$$

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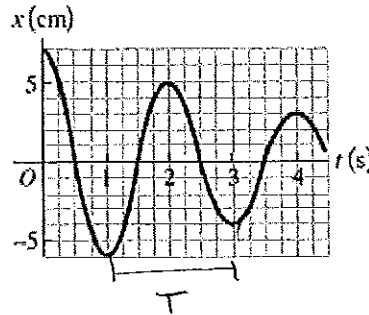
A mass is vibrating at the end of a spring of force constant 225 N/m. Figure below shows a graph of its position  $x$  as a function of time  $t$ . (a) At what times is the mass not moving? (b) How much energy did this system originally contain? (c) How much energy did the system lose between  $t=1$  s and  $t=4$  s?

$$T = 2\pi \rightarrow \omega = \frac{1}{2} \text{ (s}^{-1}\text{)}$$

$$v \leq \omega \text{ (rad/s)}$$

$$x_0 \leq x_{\text{max}}$$

$$x \leq x_{\text{max}} \cos \omega t$$



$$x(t = 2\pi) \leq x_{\text{max}}$$

$$E_0 \leq \frac{1}{2} k x_{\text{max}}^2 \leq \frac{1}{2} (225) (7)^2 \leq 5512.5 \text{ (J)}$$

$$E_1 - E_2 \leq \frac{1}{2} k [x^2(t=1) - x^2(t=3)]$$

$$\leq \frac{1}{2} \times 225 [36 - 25] = 1237.5 \text{ (J)}$$

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A damped oscillator that is subject to a sinusoidal driving force will perform oscillations with an amplitude given as:

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

where  $\omega_d$  and  $F_{max}$  are the angular frequency and amplitude of the driving sinusoidal force.

- Sketch A as a function of  $\omega_d$  indicating the values A take for  $\omega_d = 0$  and  $\omega_d \rightarrow \infty$ .
- Find an expression for the frequency,  $\omega_d$ , that will maximize A.

$$b) \frac{dA(\omega)}{d\omega} = F_{max} \frac{-1}{2} [(k - m\omega^2)^2 + b^2\omega^2]^{-\frac{3}{2}} \times [2(k - m\omega^2) \times (-2m\omega) + 2b^2\omega]$$

$$= \frac{F_{max} [2m\omega(k - m\omega^2) - b^2\omega]}{[(k - m\omega^2)^2 + b^2\omega^2]^{\frac{3}{2}}}$$

$$\frac{dA(\omega)}{d\omega} = 0 \Rightarrow \omega \neq 0 \text{ and } 2m(k - m\omega^2) - b^2 = 0 \Rightarrow \omega = \sqrt{\frac{k}{m} - \frac{b^2}{2m}}$$

$$\omega \neq 0 \Rightarrow A(\omega) = \frac{F_{max}}{k} \text{ , } \omega = \sqrt{\frac{k}{m} - \frac{b^2}{2m}} \Rightarrow A = \frac{F_{max}}{b \sqrt{\frac{k}{m} - \frac{b^2}{4m}}}$$

